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A CONSIDERATION OF A TURBULENT HEAT TRANSFER MODEL FOR FLOW IN SMOOTH AND ROUGH TUBES

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DUE TO the complex nature of turbulent flow, the analysis of associated heat- and mass-transfer processes have largely been of an empirical nature; these models are generally based on the eddy diffusivity concept. Interestingly, the foundation for a different approach to this problem has been suggested by Kolar [6]. The nucleus of Kolar's analysis is based on the surface renewal principle. Although Kolar apparently encountered difficulties in the evaluation of the mean frequency of renewal, this type model appears to provide insight not offered by other available models. Based on a reasonable formulation for the mean frequency of renewal, the adaption of this elementary model to heat transfer for turbulent flow in smooth and rough tubes will now be reconsidered.

The turbulent heat-transfer model proposed by Kolar is based on the assumption that fluid motion at the solid-fluid interface consists of a mosaic of vortex or eddy elements which are intermittently replaced by fresh fluid from the bulk stream. These elements are said to represent the actual resistance to heat transfer at the surface. Kolar further assumed that the energy transfer within individual elements at the surface may be characterized as simple one-dimensional unsteady molecular transfer,

$$\frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial y^2}. \tag{1}$$

This equation was coupled with initial and boundary conditions of the form $t(0, y) = T_i$, $t(\theta, 0) = T_0$, and $t(\theta, \infty) = T_i$; t is the instantaneous temperature profile within the eddy during its residence at the surface, θ is the instantaneous contact time, T_i is the eddy temperature at the first instant of renewal, and T_0 is the wall temperature. The parameter T_i has been set equal to the bulk stream temperature, T_b, which appears to be appropriate for fluids other than liquid metals.

Kolar accounted for the effect on the mean transport of the numerous eddies at the surface by merely assuming that all fluid elements remain in contact with the surface for the same length of time. Hence, based on this model, an expression may be written for the mean temperature profile, T, of the form

$$T = \frac{1}{\tau}\int_0^{\tau} t \, d\theta \tag{2}$$

where the mean residence time, τ, represents the mean length of time eddies remain at the surface. This analysis gives rise to an expression for the mean coefficient of heat transfer, h, of the form

$$h = \frac{2}{\sqrt{\pi}}\left(\frac{\rho c k}{\tau}\right)^{\frac{1}{2}}. \tag{3}$$

It should be observed that Kolar's turbulent heat transport model is identical to the surface renewal and penetration model. This type model was first adapted to turbulent mass-transfer processes at fluid–fluid interfaces by Danckwerts [1], and later adapted to turbulent momentum transfer associated with solid–fluid interfaces by Einstein and Li [3] and Hanratty [4]. More recently the surface renewal and penetration model has been applied to turbulent liquid metal heat transfer [9] and turbulent boundary layer flow [10].

Kolar formulated an expression for the mean frequency of renewal, $1/\tau$, which is based on the notion that wall turbulence is homogeneous and isotropic. Kolar essentially set τ equal to λ_0/V_{λ_0} where V_{λ_0} is the local velocity fluctuation and λ_0 is the local characteristic length in which velocity changes occur. After certain questionable assumptions, Kolar obtained an empirical expression for τ of the form

$$U^* \sqrt{\left(\frac{\tau}{\nu}\right)} = \frac{1}{28}. \tag{4}$$

A more rigorous formulation for the mean frequency of renewal may be obtained on the basis of momentum transfer [9]. An analysis for the mean velocity profile, u, which is synonymous to the formulation for the mean temperature profile presented earlier, leads to an expression for the mean wall shear stress, σ_0, of the form

$$\sigma_0 = \mu \left.\frac{\partial u}{\partial y}\right|_0 = \frac{2}{\sqrt{\pi}} \rho\, U_i \sqrt{\left(\frac{\nu}{\tau}\right)} \quad \text{(skin friction)} \tag{5}$$

where U_i represents the eddy velocity at the first instant of renewal. With the mean shear stress written in terms of the friction velocity U^*, an expression for τ may be written as $[U^* = \sqrt{(\sigma_0/\rho)}]$

$$U^*\sqrt{\left(\frac{\tau}{\nu}\right)} = \frac{2}{\sqrt{\pi}}\frac{U_i}{U^*}. \qquad (6)$$

This type formulation for the mean frequency of renewal was first proposed by Einstein and Li [3] and Hanratty [4]. Hanratty assumed that U_i/U^* may be equated to a constant equal to 13·5. As an alternative, it appears that U_i may be fairly well represented by the bulk stream velocity, U_b, such that U_i/U^* becomes equal to $\sqrt{(2/f)}$, where f is the Fanning friction factor.

DISCUSSION

The coupling of equation (3) with equations (4) and (6) lead to the following expressions for the local mean Nusselt number

$$Nu = 0{\cdot}0404\sqrt{\left(\frac{f}{2}\right)}Re\sqrt{Pr} \qquad (7)$$

and

$$Nu = \frac{f}{2}Re\sqrt{Pr}. \qquad (8)$$

Turbulent flow in smooth tubes

These expressions are compared with experimental heat-transfer data by Dipprey and Sabersky [2] for fully developed turbulent flow in smooth tubes in Fig. 1. Equation (8) is seen to adequately represent these data. Figure 1 indicates that the predicted proportionalities, $Nu \sim f/2$ and $Nu \sim \sqrt{Pr}$, are consistent with these experimental data. Additional evidence regarding the functional relationship between Nu and f is provided by Hubbard and Lightfoot [5]. These investigators have found that the proportionality $Nu \sim f/2$ is in agreement with experimental heat- and mass-transfer data for values of the Prandtl (Schmidt) number as large as 6000.

The surface renewal and penetration model has also been employed in the formulation of expressions for the mean temperature profile [9]. The model leads to expressions for the dimensionless temperature profile which correlate data for $0{\cdot}02 < Pr < 5{\cdot}0$. For values of the Prandtl number very much greater than unity, the elementary surface renewal and penetration model has been found to be inappropriate [8]. This result is apparently due to the effect on the heat transfer of eddies not reaching the surface.

Turbulent flow in rough tubes

The applicability of the surface renewal and penetration model will now be considered for flow over rough surfaces. Importantly, the formulation for τ presented herein is based on a fairly concise physical picture of the renewal process. This more complete modeling of the renewal process will now be helpful in considerations regarding flow in rough tubes.

Based on the adaptation of the surface renewal and penetration model to momentum transfer, the expression for the mean wall shear stress, equation (5), accounts for

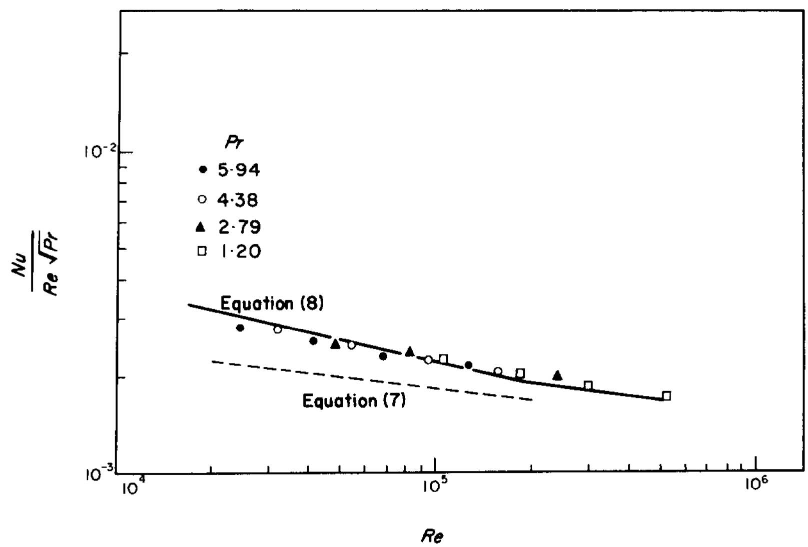

FIG. 1. Heat transfer and friction factor data (smooth tube) by Dipprey and Sabersky [2].

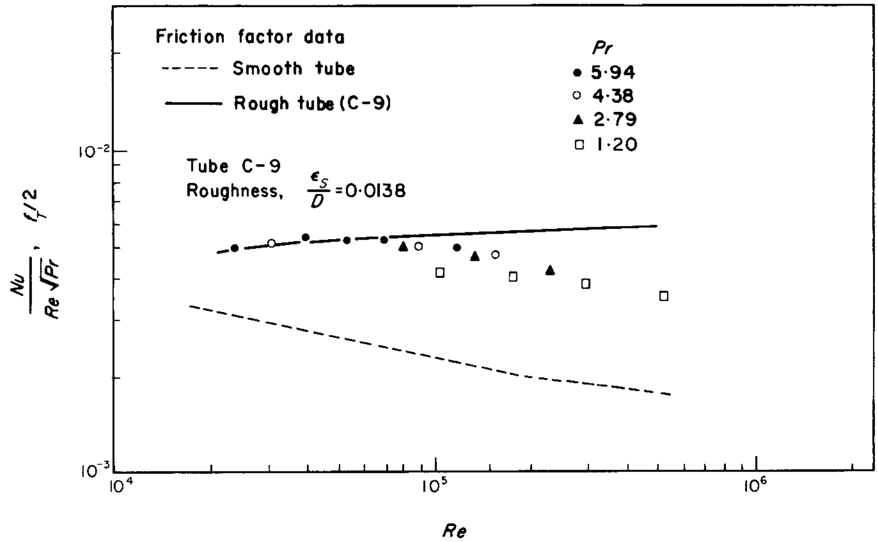

FIG. 2. Heat transfer and friction factor data (rough tube, C-9) by Dipprey and Sabersky [2].

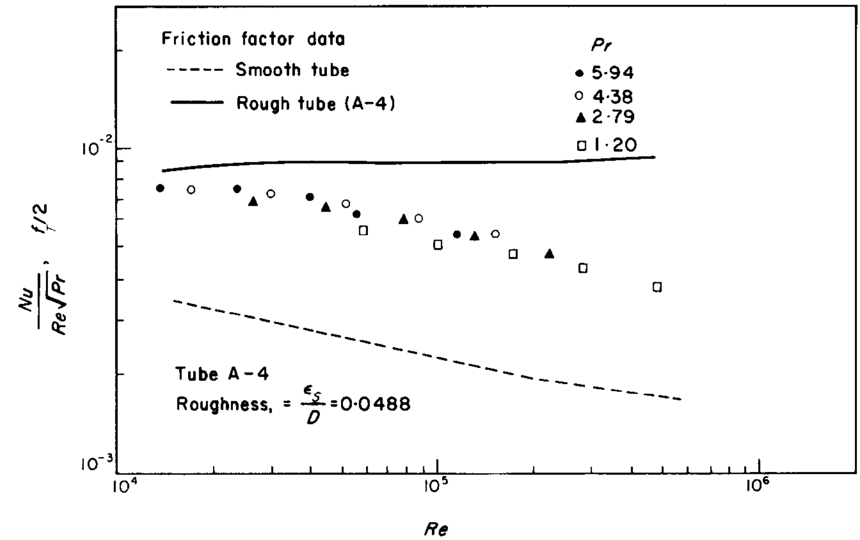

FIG. 3. Heat transfer and friction factor data (rough tube A-4) by Dipprey and Sabersky [2].

skin friction only. Accordingly, the expression for heat transfer given by equation (8), and for that matter equation (7), fails to account for the effect of form drag; i.e. the friction factor in these equations represents the skin friction only. Due to the significance of form drag, this consideration becomes quite important in the study of heat transfer for flow over rough surfaces.

Figures 2 and 3 illustrate Dipprey and Sabersky's heat transfer and friction factor data for turbulent flow in rough tubes; the apparent friction factor, f_T, accounts for the effect on the pressure drop of both form drag and skin friction. It is apparent that the significance of form drag increases for increasing values of the relative roughness, ε_s/D. It is observed in Figs. 2 and 3 from [7] that for fully rough flow the apparent friction factor is independent of the Reynolds number as a result of the predominance of the form drag. Interestingly, these figures indicate that the dimensionless group $Nu/(Re\sqrt{Pr})$ becomes proportional to f (smooth) for fully rough flow.

Kolar compared experimental data for both smooth and rough tubes with an adjusted expression of the form

$$Nu = 0{\cdot}0517\sqrt{\left(\frac{f}{2}\right)}Re\sqrt{Pr}. \tag{9}$$

The empirical adjustment of equation (7) was introduced by Kolar in order to compensate for the uncertainty in his formulation for τ. These experimental data were found by Kolar to seriously diverge from equation (9) for increasing values of the parameter $\sqrt{(f_T/2)}\cdot Re$. Accordingly, Kolar suggested the necessity of correcting these data due to thermocouple errors. Kolar assumed that the data for higher values of $\sqrt{(f_T/2)}\cdot Re$ diverge from equation (9) because the thermocouples inserted into the tube wall do not indicate the surface temperature of the roughness element, but rather approximate the temperatures at the root. Kolar recalculated his heat-transfer data for $\sqrt{(f_T/2)}\cdot Re > 3 \times 10^3$, assuming that the roughness element may be considered as a straight fin. According to Kolar, the recalculated data are well correlated by the theoretical model. However, a more logical explanation for the behavior of Kolar's raw data lies in a consideration of the effect of form drag. Further, it should be recalled that experimental data for smooth tubes indicate that the Nusselt number is better correlated by $f/2\cdot Re$ than by $\sqrt{(f/2)}\cdot Re$.

CONCLUDING REMARKS

The usefulness of the simple surface renewal and penetration model has been demonstrated for turbulent flow in smooth tubes. The successful formulation of an expression for the mean Nusselt number is dependent upon the analogous mechanisms of heat transfer and momentum transfer resulting from skin friction. However, consideration of heat transfer data for flow over rough surfaces suggests the significance of form drag. Accordingly, the adaptation of this type model to turbulent flow processes in which the form drag is predominant, such as flow past submerged bodies as well as flow over rough surfaces, will apparently require additional information regarding the mean frequency of renewal.

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